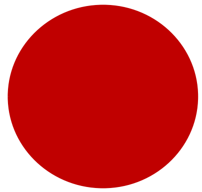
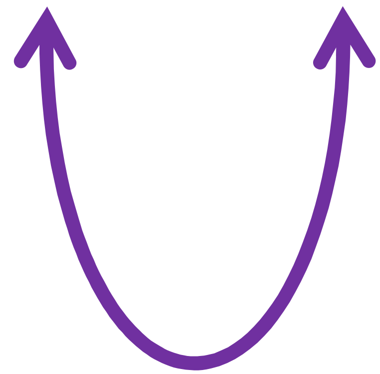
Ch. 11 Conics



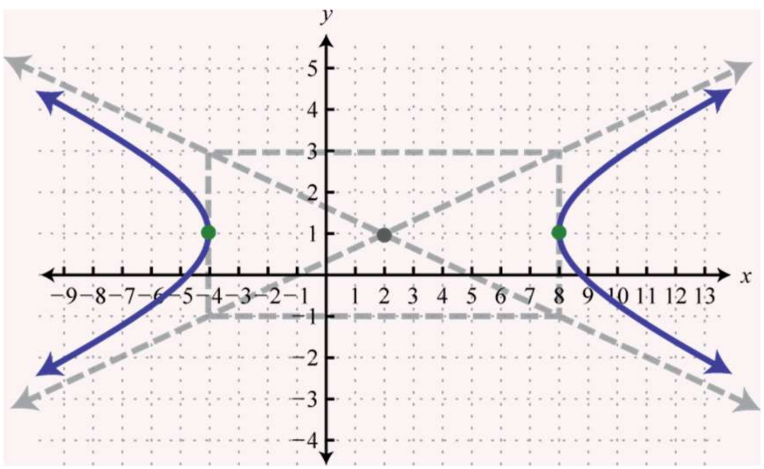


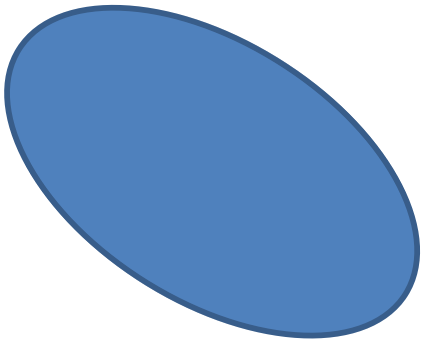
Circle

Parabola

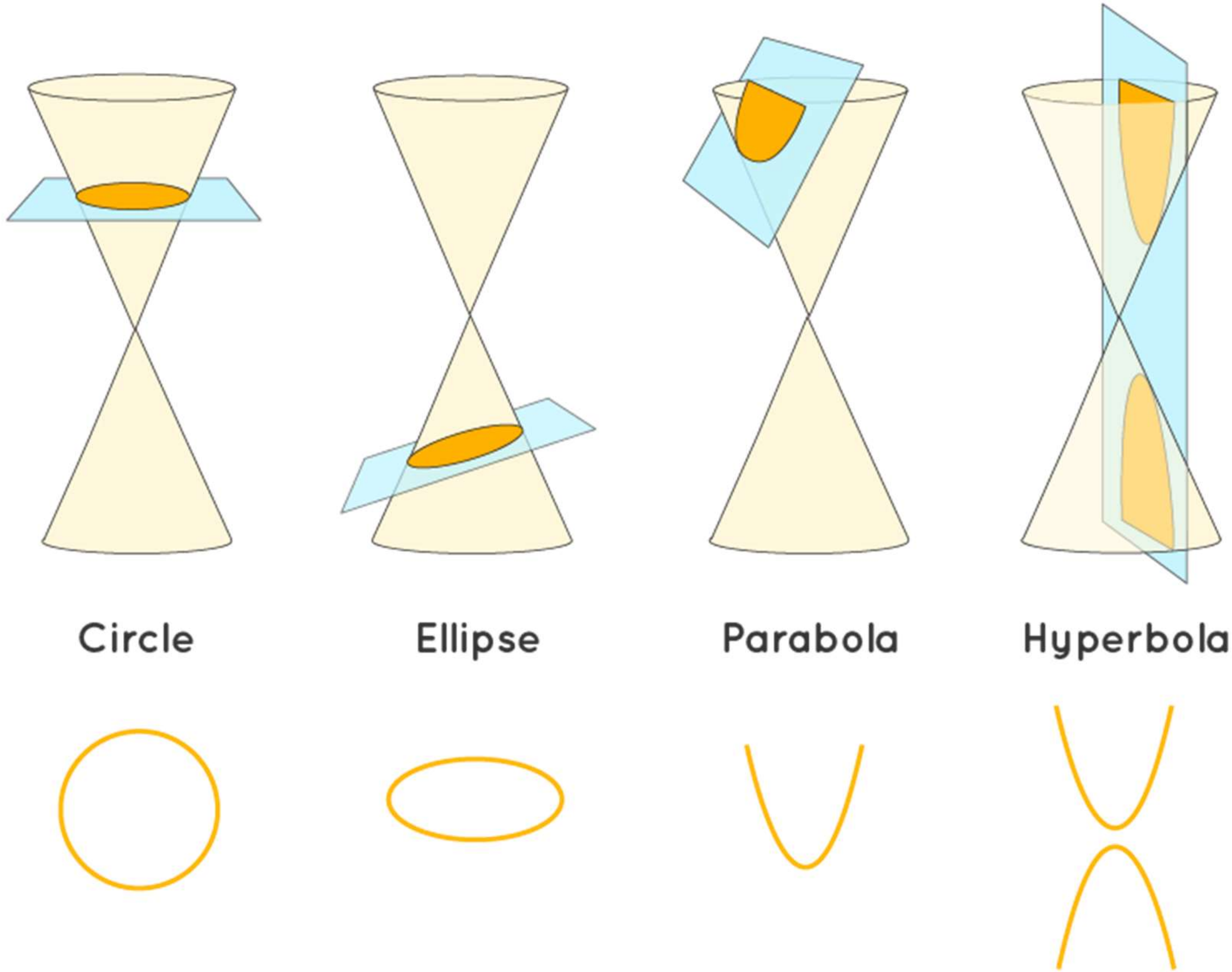
Ellipse

Hyperbola





How the 4 conic sections are formed:



Add notes to pink sheet as needed:

Polar Coordinates

$r^2 = x^2 + y^2$ or $r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x} \sin^{-1}\theta$

$x = r\cos\theta$

$y = r\cos\theta$

polar form of a complex number
$r(\cos\theta + i\sin\theta)$

$z_1 \bullet z_2 =$
$r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

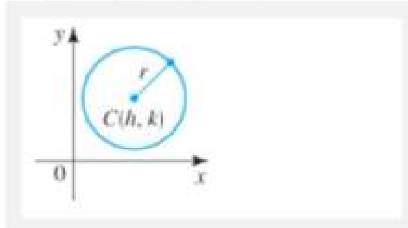
$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

De...re's Theorem
$[r(\cos\theta \ldots in\theta)]^n$
$= r \ldots s\, n\theta + i\sin n\theta)$

Conic Sections

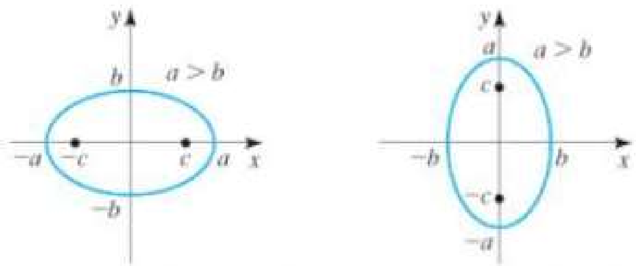
Circles

$(x-h)^2 + (y-k)^2 = r^2$



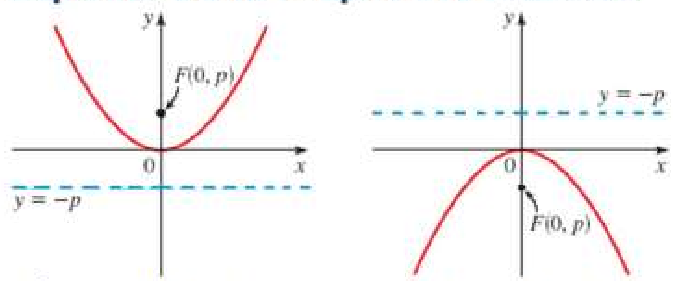
Ellipses

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

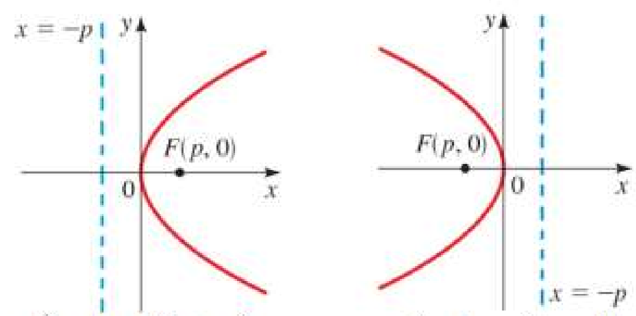


Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$ Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

Equations and Graphs of Parabolas



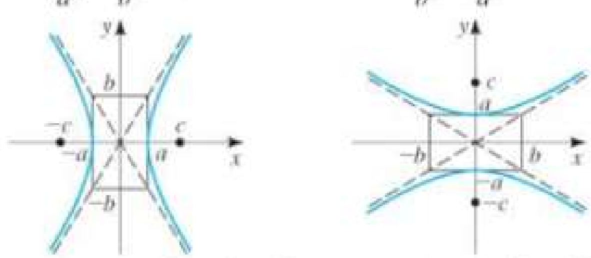
$x^2 = 4py$ with $p > 0$ $x^2 = 4py$ with $p < 0$



$y^2 = 4px$ with $p > 0$ $y^2 = 4px$ with $p < 0$

Hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $-\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$ Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

Hyperbola equations (on your formula sheet)

Note: The positive term with a^2 dictates the orientation of the hyperbola.

Note: a is on the transverse axis. It can be shorter or longer than b.

positive term horizontal

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

positive term vertical

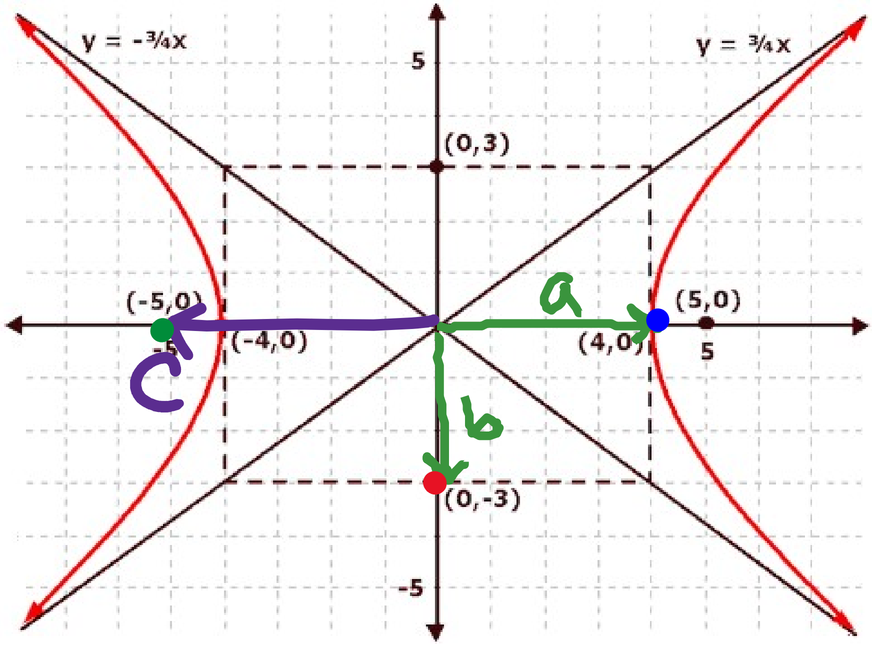
$$-\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

or

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

positive term vertical

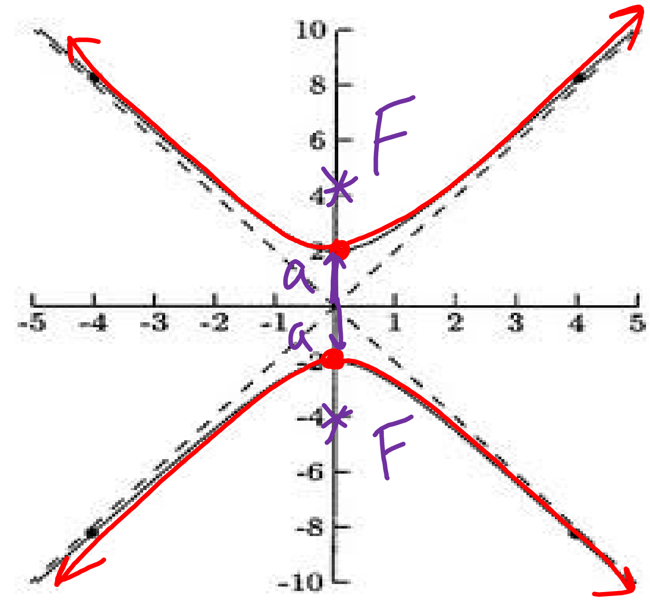
Horizontal example:



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

positive term horizontal

Vertical example:



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

positive term vertical

Other notes: Hyperbolas

vertices: located "a" units from the center.

a is always with the positive term

transverse axis: the line connecting the two vertices (length = 2a)

foci: located on the transverse axis, "c" units from the center foci: located on the transverse axis, "c" units from the center

see formula sheet → $c^2 = a^2 + b^2$

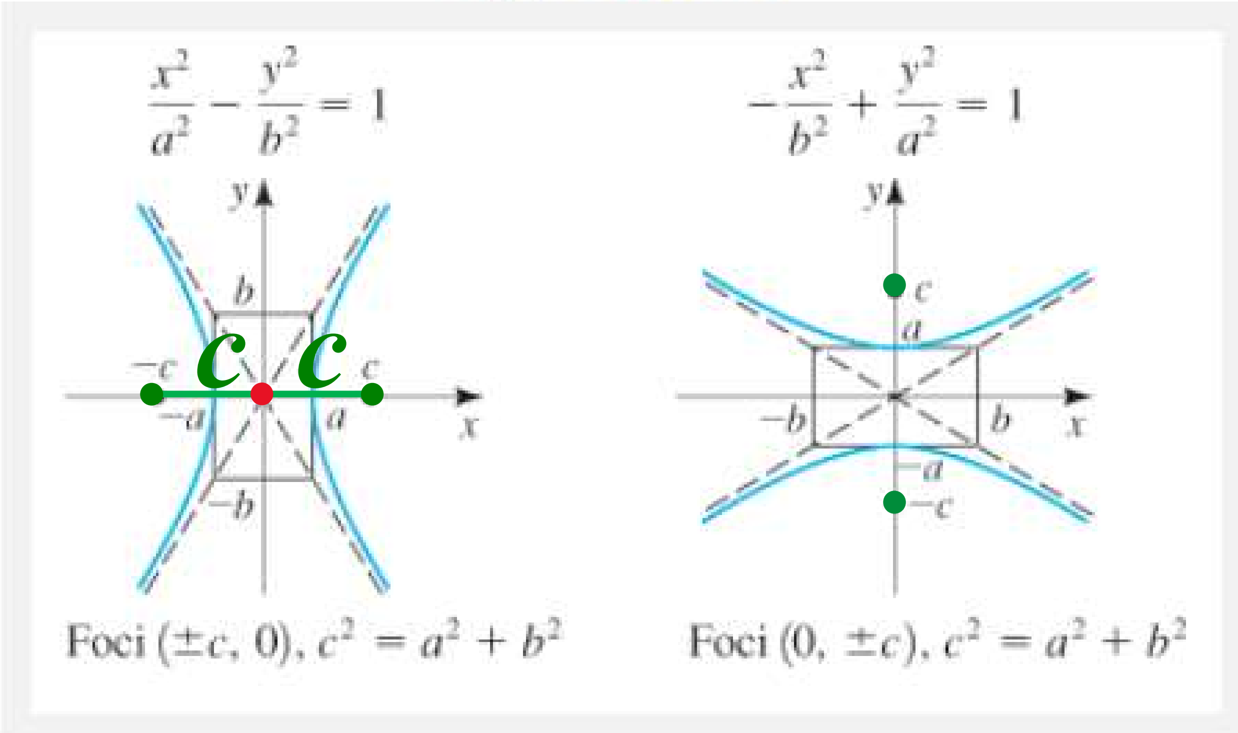
to find foci of hyberbola

add

(compare to an ellipse → $c^2 = a^2 - b^2$*)*

subtract

The **foci** are always located "inside" the curves.



$$c^2 = a^2 + b^2$$

The asymptotes are the diagonals of the central box.

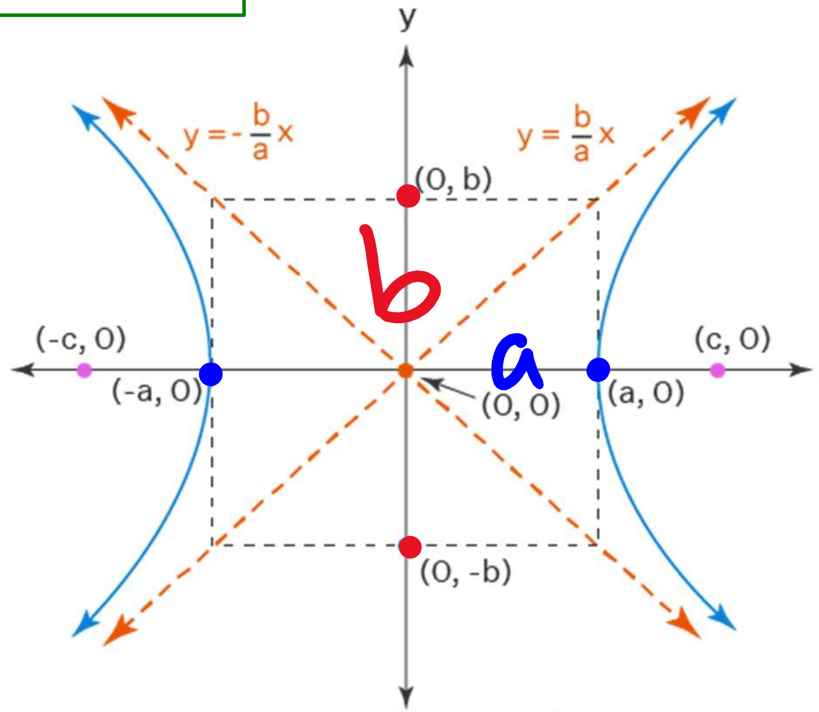
$$y = \pm \frac{b}{a} x$$

horizontal

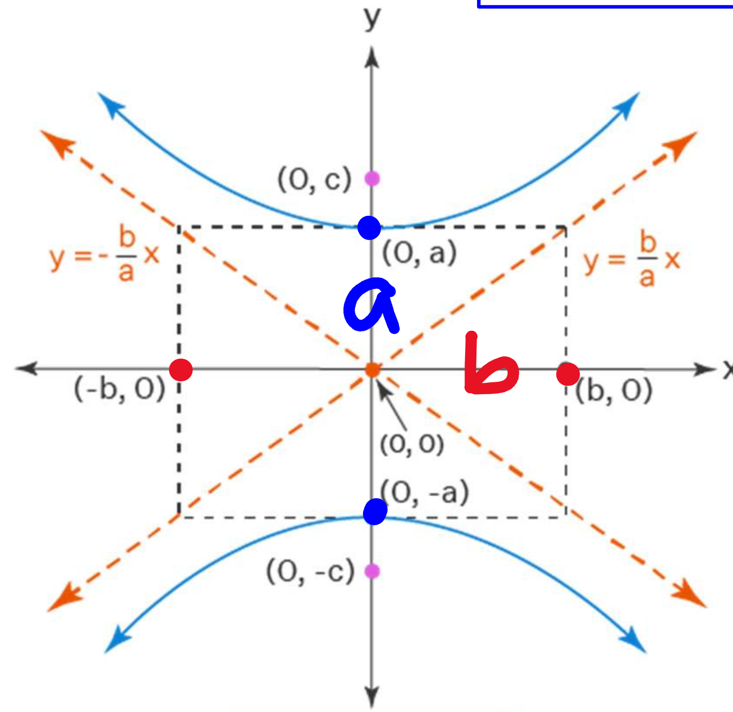
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$y = \pm \frac{a}{b} x$$

vertical



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



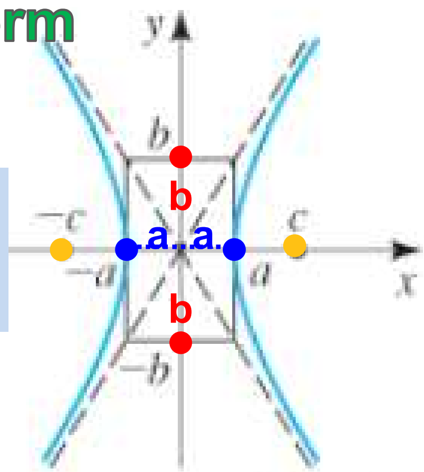
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$y = \pm \frac{b}{a} x$ **Asymptotes** $y = \pm \frac{a}{b} x$

Horizontal orientation because x term is positive

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

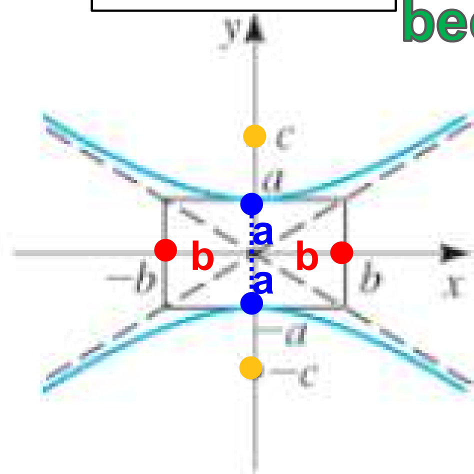
transverse axis = 2a



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$

Vertical orientation because y term is positive

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

2 vertices always at ends of the transverse axis

$$c^2 = a^2 + b^2$$

2 foci located on transverse axis "c" units from the center

Add hyperbola notes to pink sheet!

Label the vertices, foci, and asymptotes for the given graphs:

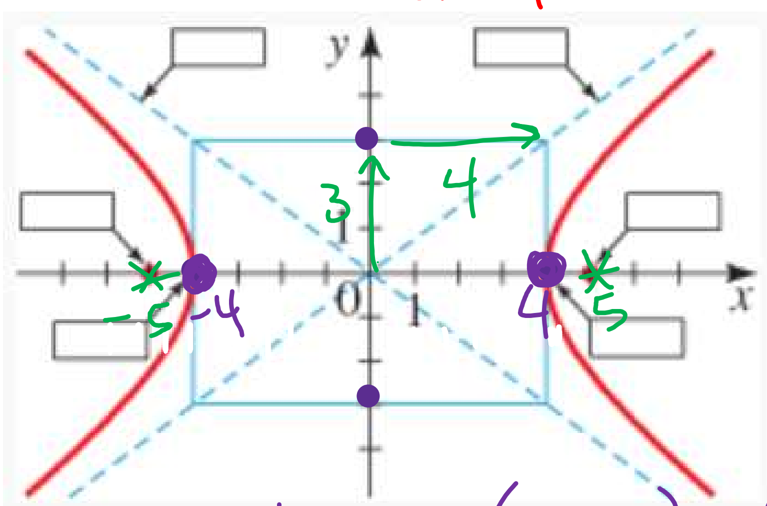
4. (a) $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$ (b) $\frac{y^2}{4^2} - \frac{x^2}{3^2} = 1$

horiz

a=4   b= 3

vertical

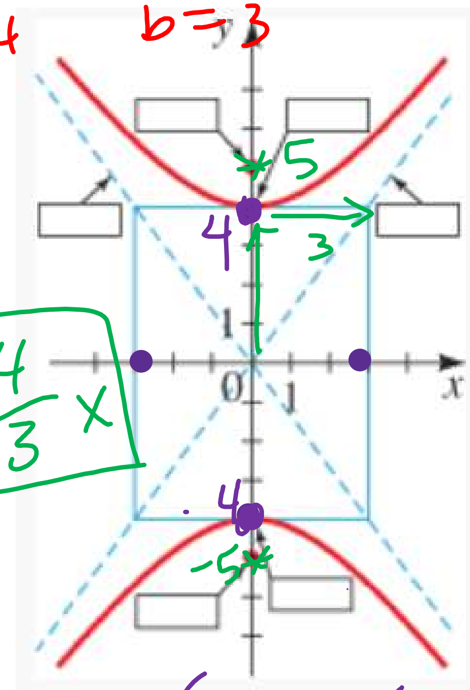
a=4   b=3



$y = \pm\frac{3}{4}x$

asymptotes

$y = \pm\frac{4}{3}x$



Vertices (-4,0), (4,0)

foci (-5,0), (5,0)

Vertices (0,4), (0,-4)

foci (0,5) (0,-5)

#5-8: Match the equations with the graphs

(write equation, show work!)

5. $\frac{x^2}{4} - \frac{y^2}{1} = 1$

horizontal orientation

$a^2 = 4$

$a = 2$ (vertices)

III

6. $\frac{y^2}{1} - \frac{x^2}{9} = 1$

vertical

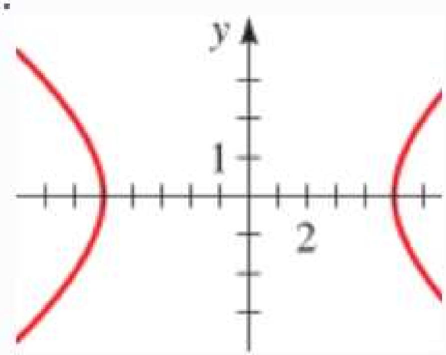
a^2 $a = 1$

IV

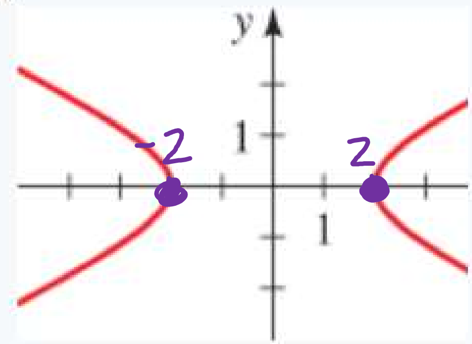
7. $16y^2 - x^2 = 144$

8. $9x^2 - 25y^2 = 225$

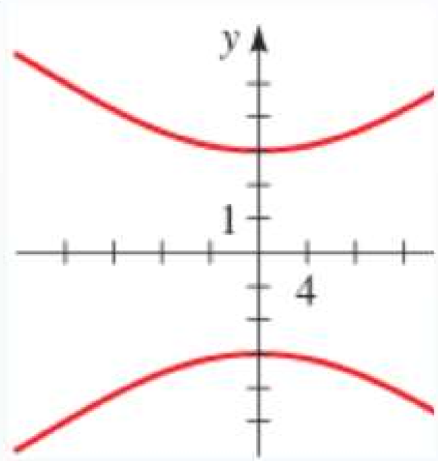
I.



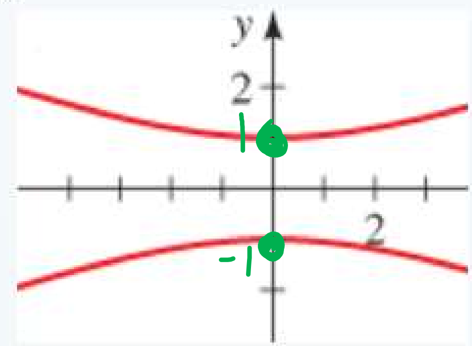
III.



II.



IV.



#5-8: Match the equations with the graphs (write equation, show work!)

5. $\dfrac{x^2}{4} - y^2 = 1$

6. $y^2 - \dfrac{x^2}{9} = 1$

7. $16y^2 - x^2 = 144$

$\dfrac{16y^2}{144} - \dfrac{x^2}{144} = \dfrac{144}{144} \rightarrow \dfrac{y^2}{9} - \dfrac{x^2}{12} = 1$

vertical

$a^2 \rightarrow a = 3$

II

8. $9x^2 - 25y^2 = 225$

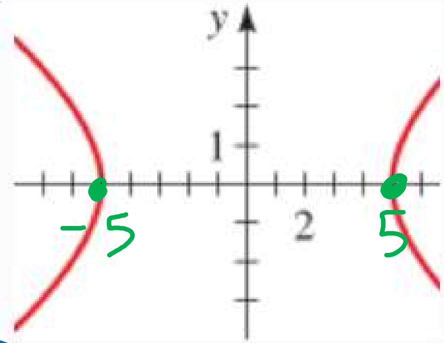
$\dfrac{9x^2}{225} - \dfrac{25y^2}{225} = \dfrac{225}{225}$

horizontal $\rightarrow \dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$

$a^2 = 25$
$a = 5$

I

I.



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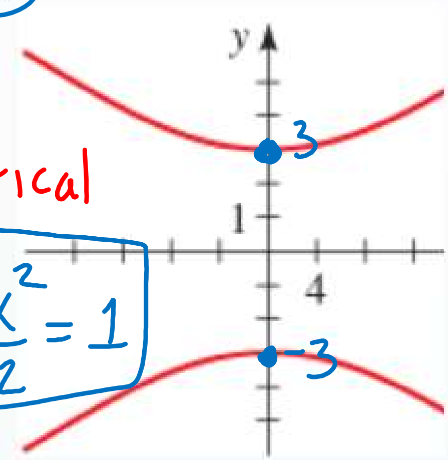
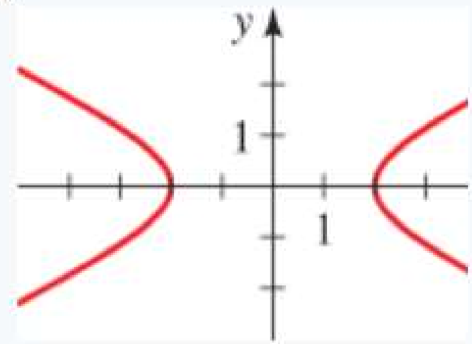
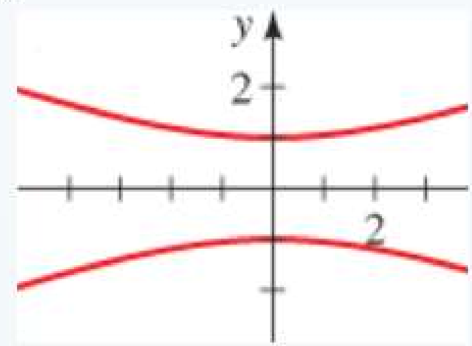


III.



IV.



#9, 10, 17-23odd

(b) Find the vertices, foci, and asymptotes. $(\pm 2, 0)$ $y = \pm \frac{4}{2}x$ $y = \pm 2x$

(c) Determine length of the transverse axis. $= 2(a) = 2(2) = 4$

(a) Sketch using the central box method.

horizontal

$$9.\ \frac{x^2}{4} - \frac{y^2}{16} = 1$$

$a^2 = 4$ $b^2 = 16$

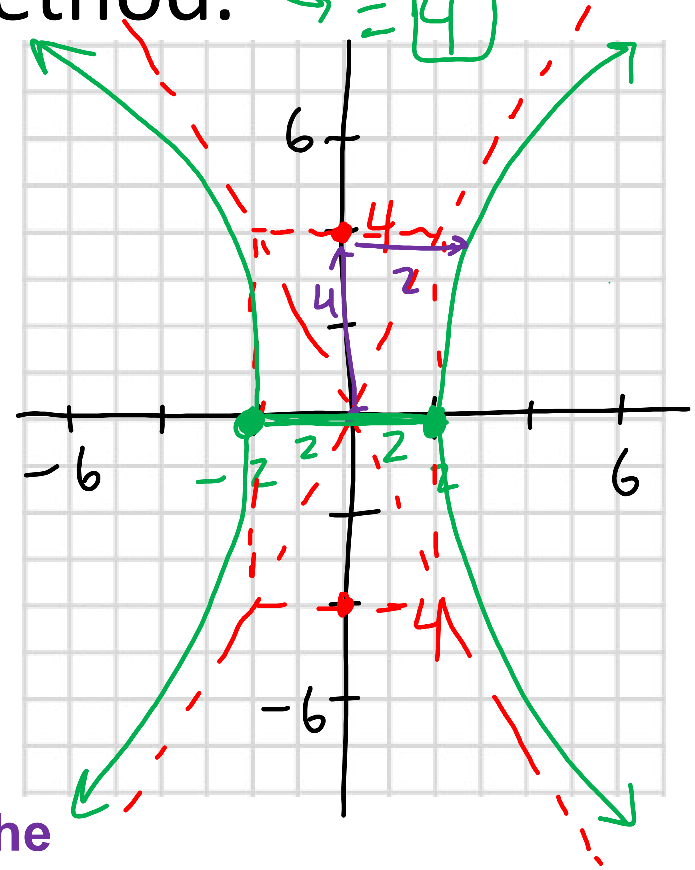
$a = 2$ $b = 4$

Graph by plotting points using the a and b values.

Create central box and extend diagonals.

Sketch curves using the vertices and asymptotes.

Then identify vertices, foci, etc…



See foci on next slide ↓

#9, 10, 17-23odd

(b) Find the vertices, foci, and asymptotes.

(c) Determine length of the transverse axis.

(a) Sketch using the central box method.

vertices: $(\pm 2, 0)$

asymptotes: $y = \pm \frac{4}{2}x$, $y = \pm 2x$

transverse axis $= 2(a) = 2(2) = 4$

horizontal

9. $$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

$a^2 = 4$, $b^2 = 16$

$a = 2$, $b = 4$

$c^2 = a^2 + b^2$

$c^2 = 2^2 + 4^2$

$c^2 = 4 + 16$

$c^2 = 20$

$c = \sqrt{20}$ or $2\sqrt{5}$

foci $(\pm 2\sqrt{5}, 0)$